SOLUTIONS TO SELECTED QUESTIONS IN HOMEWORK 3

MATH 241

17.6.11

Proof. $e^{1+5\pi i/4}e^{-1-\pi i/3} = e^{1+5\pi i/4-1-\pi i/3} = e^{\frac{11\pi i}{12}} = \cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}$. □

17.6.25

Proof. $\ln(-2+2i) = \ln(\sqrt{(-2)^2+2^2}) + i(\frac{3\pi}{4}+2n\pi) = \frac{3}{2}\ln 2 + i(\frac{3\pi}{4}+2n\pi)$. □

17.6.37

Proof. $e^{z-1} = -ie^2 \Rightarrow e^{z-1} = e^{-\frac{\pi}{2}i}e^2 \Rightarrow e^{z-1} = e^{-\frac{\pi}{2}i+2}$. So $z-1 = -\frac{\pi}{2}i + 2 + 2n\pi i$, $z = -\frac{\pi}{2}i + 3 + 2n\pi i$. □

17.6.41

Proof. $(1+i)^{1+i} = e^{(1+i)\ln(1+i)} = e^{(1+i)(\ln\sqrt{2}+\frac{\pi i}{4}+2n\pi i)} = e^{\ln\sqrt{2}-\frac{\pi}{4}-2n\pi+i(\ln\sqrt{2}+\frac{\pi}{4}+2n\pi)} = \sqrt{2}e^{-\frac{\pi}{4}-2n\pi}(\cos(\ln\sqrt{2}+\frac{\pi}{4}) + i\sin(\ln\sqrt{2}+\frac{\pi}{4}))$. □

17.7.19

Proof. $\cos z = \sin z$ is equivalent to $e^{iz} - e^{-iz} = i(e^{iz}+e^{-iz})$. So e^{iz} is the root for $(1-i)t^2 = 1+i$, so $t^2 = i$, $t = \pm(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i)$, $iz = i\frac{\pi}{4} + n\pi i$, $z = \frac{\pi}{4} + n\pi$. □

17.7.22

Proof. $\sin z = i\sinh 2$, e^{iz} is the root for $t^2 + 2(\sinh 2)t - 1 = 0$. $t = -\sinh 2 \pm \sqrt{\sinh^2 2 + 1} = \pm\cosh 2 - \sinh 2 = e^{-2}$ or $-e^2$. Therefore $iz = -2 + 2n\pi i$, $z = 2i + 2n\pi$; or $iz = 2 + i(\pi + 2n\pi)$, $z = -2i + \pi + 2n\pi$. □

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Proof.

$$\cos(2i\ln(\frac{1-i}{1+i})) = \cos(2i\cdot\ln i) = \cos(2i(\frac{\pi i}{2})) = \cos(-\pi) = -1$$

$$\arcsin[(\frac{\sqrt{3}+i}{\sqrt{3}-i})^{12}] = \arcsin[(\frac{1+\sqrt{3}i}{2})^{12}] = \arcsin 1 = \frac{\pi}{2}$$

$$i^{\ln i} = e^{(\ln i)^2} = e^{(\frac{pi}{2})^2} = e^{\frac{\pi^2}{4}}$$

□